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Tree ghost in spinor gravity

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Abstract. We evaluate the residue at the graviton ghost pole for spinor–spinor scattering of gravitating spin-$\frac{1}{2}$ particles for the lowest-order graph, and prove it to be non-zero. We conclude that attempts to quantize gravity in models for which counter terms are required are unsuccessful; a possible class of models is briefly considered.

1. Introduction

The counter terms needed to renormalize a quantum field theoretic version of Einstein's gravitational Lagrangian were shown earlier (Nouri-Moghadam and Taylor 1976a, b, to be referred to as I and II) to produce ghost particles at the tree graph level which could not be removed by a suitable choice of parameters both for scalar and vector matter fields. This disturbing situation requires further clarification, both by investigating higher-order graphs and also by considering other sources of the gravitational field. We turn to the latter question in this paper, in particular considering the case of a world composed purely of spin-$\frac{1}{2}$ particles interacting through their gravitational field; the question of higher orders we defer to another publication.

It could be argued that our earlier results (Nouri-Moghadam and Taylor 1975) and those of other workers (t'Hooft and Veltman 1974, Deser and van Nieuwenhuizen 1974a, b, Capper and Duff 1974) already indicate the lack of renormalizability of any quantum field theory including Einstein's non-linear Lagrangian $R\sqrt{-g}$. However we feel that if it were possible, by a suitable choice of matter terms, to eliminate the ghosts arising from the one-loop counter terms then a possible model for further investigation at the two-loop level would have been obtained. Since there is clearly a very important question at stake, namely quantizing Einsteinian gravity, then we should continue our earlier quest for ghost elimination as energetically as possible. This paper is devoted to that quest.

In the next section we formulate the problem in terms of the effective vertex functions for the spinors and propagator for the graviton. We discuss the ghost pole contributions in the next section and determine the range of conditions under which this can be eliminated. We conclude with a brief discussion.

2. The vertex and propagator

The notation will be the same as in I and II, so that the graviton propagator $\langle T(g_{\mu\nu}g_{\lambda\sigma})\rangle_0$

for the graviton field $g_{\mu\nu}$ arising from the Lagrangian

$$L_g=\sqrt{-g}(R+aR^2+4bR_{\mu\nu}R^{\mu\nu})$$

is

$$Q_{\mu\nu\lambda\sigma}=\sum_{i=1}^{6}A_iT_i$$

where the T_i are the six independent fourth-rank tensors of I and the A_i are the functions of the graviton mass p^2 only with poles at $p^2=0$, $p^2=1/2(3a+4b)$, and $p^2=1/4b$, and are given in I.

The matter Lagrangian for the self-gravitational interaction of a spin-$\frac{1}{2}$ neutrino and field operator ψ will be

$$L_{\text{matter}}=\sqrt{-g}\,\mathrm{i}L^{\mu a}\bar{\psi}\gamma_\mu\tfrac{1}{4}(1+\gamma_5)(\vec{\nabla}_\mu-\overleftarrow{\nabla}_\mu)\psi \tag{2.1}$$

where $L^{\mu a}$ is the *vierbein* and the γ are the usual Dirac matrices; the arrows on the covariant differentiation symbol ∇_μ denote the direction it acts. In terms of the expansion about the Lorentz metric $\eta_{\mu\nu}$

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$
$$\sqrt{g}=(1+\tfrac{1}{2}h^{\alpha}{}_{\alpha}+\ldots)$$
$$L^{\mu a}=\eta^{\mu a}-\tfrac{1}{2}h^{\mu a}+\ldots$$

we have that the Lagrangian (2.1) becomes

$$\mathrm{i}(1+\tfrac{1}{2}h^{\alpha}{}_{\alpha})(\eta^{\mu a}-\tfrac{1}{2}h^{\mu a})\bar{\psi}\gamma_a\tfrac{1}{4}(1+\gamma_5)(\vec{\partial}_\mu-\tfrac{1}{4}B_{\mu[\alpha\beta]}\sigma^{\alpha\beta}\ldots)\psi$$

where $B_{\mu[\alpha\beta]}$ is the spinor connection defined in Salam and Strathdee (1970), and the resulting vertex function for the emission of a graviton $h_{\alpha\beta}$ from the spinor particle with change of momentum from k_1 to $-k_2$ of figure 1 will thus be

$$V_{\alpha\beta}(k_1,k_2)=\tfrac{1}{16}(2g_{\alpha\beta}g^{\mu a}-g^{\mu}_{\alpha}g^{a}_{\beta}-g^{\mu}_{\beta}g^{a}_{\alpha})\gamma_a(1+\gamma_5)(k_1-k_2)_\mu. \tag{2.2}$$

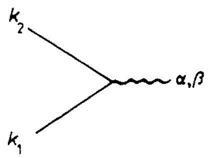

Figure 1. The vertex function $V_{\alpha\beta}(k_1,k_2)$ determining the process of pair annihilation to produce a graviton of polarization α,β.

The vertex function has now to be applied to both ends of the graviton propagator in order to evaluate the graph of figure 2. The result will thus be

$$V_{\alpha\beta}(k_1,k_2)Q^{\alpha\beta\mu\nu}V_{\mu\nu}(k_3,k_4). \tag{2.3}$$

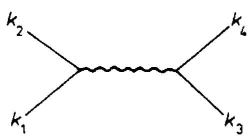

Figure 2. The annihilation–creation process with a single intermediate graviton giving a ghost contribution.

This can be evaluated after some calculation to be

$$
\begin{aligned}
&\tfrac{1}{256}\gamma_a^{(1)}(1+\gamma_5^{(1)})(k_1-k_2)_\lambda\gamma_{a'}^{(1)}(1+\gamma_5^{(2)})(k_3-k_4)_{\lambda'}[36A_1 g^{\lambda a}g^{\lambda' a'} \\
&\quad +2A_2(g^{\lambda\lambda'}g^{aa'}+g^{\lambda a'}g^{\lambda' a}+g^{\lambda a}g^{\lambda' a'})+A_3(24\Box g^{\lambda a}g^{\lambda' a'}-12g^{\lambda a}\partial^{\lambda'}\partial^{a'} \\
&\quad -12g^{\lambda' a'}\partial^{\lambda}\partial^{a})+4A_5(\partial^\lambda\partial^a\partial^{\lambda'}\partial^{a'}-\Box g^{\lambda a}\partial^{\lambda'}\partial^{a'}-\Box g^{\lambda' a'}\partial^\lambda\partial^a+\Box^2 g^{\lambda a}g^{\lambda' a'}) \\
&\quad +A_6(g^{\lambda\lambda'}\partial^a\partial^{a'}+g^{\lambda a'}\partial^a\partial^{\lambda'}+g^{\lambda' a}\partial^\lambda\partial^{a'}+g^{aa'}\partial^\lambda\partial^{\lambda'}-4g^{\lambda' a'}\partial^\lambda\partial^a \\
&\quad -4g^{\lambda a}\partial^{\lambda'}\partial^{a'}+4\Box g^{\lambda a}g^{\lambda' a'})] \qquad (2.4)
\end{aligned}
$$

where the superscripts 1 and 2 denote the particles to which the γ matrices are attached.

3. The pole contribution

We now evaluate the expression

$$\bar{u}(k_3)\bar{v}(k_2)V_{\alpha\beta}(k_1, k_2)Q^{\alpha\beta\mu\nu}V_{\mu\nu}(k_3, k_4)v(k_4)u(k_1) \qquad (3.1)$$

with all external particles on mass-shell, $k_i^2=m^2$, $i=1$–4, where we take the spinors to have mass m at this stage and let m go to zero at the end of the calculation. Our results will be of use to discuss the massive spinor case shortly.

We use the Dirac equations

$$\not{K}_1 u(k_1)=mu(k_1) \qquad \bar{v}(k_2)\not{K}_2=-m\bar{v}(k_2)$$

where $\not{K}=\gamma_\mu k^\mu$. Then (3.1) reduces to

$$\tfrac{1}{256}\bar{u}(k_3)\bar{v}(k_2)\Gamma v(k_4)u(k_1)$$

with

$$
\begin{aligned}
\Gamma=&\,36(2m)^2A_1+2A_2(X+4m^2)+24(2m)^2(k_1+k_2)^2A_3+4(2m)^2[(k_1+k_2)^2]^2A_5 \\
&+A_6[Y+4(2m)^2(k_1+k_2)^2] \qquad (3.2)
\end{aligned}
$$

and

$$
\begin{aligned}
X=&\,(k_1-k_2)(k_3-k_4)[\gamma_a(1+\gamma_5)]^{(1)}[\gamma^a(1+\gamma_5)]^{(2)} \\
&+[(\not{K}_3-\not{K}_4)(1+\gamma_5)]^{(1)}[(\not{K}_1-\not{K}_2)(1+\gamma_5)]^{(2)} \\
Y=&\,(k_1-k_2)(k_3-k_4)[\not{p}(1+\gamma_5)]^{(1)}[\not{p}(1+\gamma_5)]^{(2)}.
\end{aligned}
$$

We may rewrite Y immediately as

$$4m^2(k_1-k_2)(k_3-k_4)\gamma_5^{(1)}\gamma_5^{(2)}$$

whilst the second term in X can be written in terms of the usual covariants for N–N scattering (Scadron and Jones 1968) by means of the vectors

$$P=\tfrac{1}{2}(k_1+k_3) \qquad Q=\tfrac{1}{2}(k_2+k_4) \qquad \Delta=(k_3-k_1)$$

$$\gamma^{(2)}(k_1-k_2)=\gamma^{(2)}(4P-2m) \qquad \gamma^{(1)}(k_3-k_4)=\gamma^{(1)}(2m-4Q)$$

so that

$$[(\not{K}_1-\not{K}_2)(1+\gamma_5)]^{(2)}[(\not{K}_3-\not{K}_4)(1+\gamma_5)]^{(1)}$$
$$=-16[\not{P}(1+\gamma_5)]^{(2)}[\not{Q}(1+\gamma_5)]^{(1)}+8m(1-2\gamma_5)^{(2)}[\not{Q}(1+\gamma_5)]^{(1)}$$
$$+8m(1-2\gamma_5)^{(1)}[\not{P}(1+\gamma_5)]^{(2)}-4m^2(1-2\gamma_5)^{(1)}(1-2\gamma_5)^{(2)}.$$

The parity and time-reversal conserving parts of this last expression is (Goldberger *et al* 1957):

$$-16[\not{Q}^{(1)}\not{P}^{(2)}+(\not{Q}\gamma_5)^{(1)}(\not{P}\gamma_5)^{(2)}]+8m(\not{Q}^{(1)}+\not{P}^{(2)})-4m^2(1+4\gamma_5^{(1)}\gamma_5^{(2)})$$

and we may use the equivalence identities (Scadron and Jones 1968) to write this latter expression as

$$+24\nu(K_S-K_P)+K_T(16P^2-2t)-4m^2K_S-16(P^2+\nu-\tfrac{1}{2}m^2)K_V+(4t-16\nu)K_A$$

in terms of the five covariants $K_S=1$, $K_P=\gamma_5^{(1)}\gamma_5^{(1)}$, $K_A=(\gamma_\mu\gamma_5)^{(1)}(\gamma_\mu\gamma_5)^{(2)}$, $K_T=\sigma_{\mu\nu}^{(1)}\sigma_{\mu\nu}^{(2)}$, $K_V=\frac{1}{2}\gamma_\mu^{(1)}\gamma_\mu^{(2)}$, where $\nu=PQ$, and t is the usual invariant Δ^2.

If we consider the limit as $m\to 0$ in the parity and time-reversal conserving part of the amplitude Γ we obtain on that same part D of the term X. When $m=0$, the covariant K_T depends on other covariants as

$$\tfrac{1}{4}tK_T=(K_S-K_P).$$

Substituting this and evaluating the terms in X as functions of the centre-of-mass momentum k and the scattering angle θ we obtain

$$X=4k^2(s\cos\theta-4)(K_V+K_A).$$

Thus the graviton pole contribution is proportional to

$$(s\cos\theta-4)/(k^2+1/16b).$$

This cannot be made zero by any choice of b except $b=0$, when the whole approach using counter terms breaks down (Nouri-Moghadam and Taylor 1976a). Thus the graviton pole cannot be removed from tree graphs involving neutrinos as external particles.

If we turn to the massive case, we consider the Lagrangian (2.1) without γ_5 with the added mass term $m\sqrt{-g}\bar{\psi}\psi$. This gives an addition to the graviton–two-spinor vertex $V_{\alpha\beta}(k_1,k_2)$ of (2.2) equal to $\frac{1}{2}mg_{\alpha\beta}$, and so gives the two further contributions to the single-graviton tree graph (2.3) equal to

$$\tfrac{1}{4}m^2Q^{\alpha\alpha\mu\mu}+\tfrac{1}{2}mQ^{\alpha\alpha\mu\nu}V_{\mu\nu}(k_3,k_4)+\tfrac{1}{2}mV_{\alpha\beta}(k_3,k_4)Q^{\alpha\beta\mu\mu}.$$

But $Q^{\alpha\alpha\mu\nu}=Q^{\alpha\beta\mu\mu}=0$ at the graviton pole at $p^2=-1/4b$ (Nouri-Moghadam and Taylor 1976a), so these additional terms give no contribution to our discussion. We are thus left with the amplitude Γ of (3.2) with γ_5 deleted everywhere from it. The reduction to the covariants K_S, K_P, K_V, K_A and K_T can now proceed, these being independent. The result of this reduction for the coefficient of K_T is $-tA_2/64$. The graviton pole contribution in the centre of mass of incoming particles thus is proportional to

$$(\tfrac{1}{4}p^2-m^2)(1-\cos\theta)/bp^2(p^2+1/4b).$$

We can remove the pole at $p^2 = -1/4b$ if $m^2 = -1/4b$; the coefficient of K_P is proportional to

$$
\begin{aligned}
(16m - 8\nu)A_2 &= \tfrac{1}{4}(k + k', 2k_0)(-k - k', 2k_0) \\
&= \tfrac{1}{4}[4k_0^2 + (k + k')^2] = \tfrac{1}{4}p^2 + \tfrac{1}{4}(k + k')^2 \\
&= -1/16b + \tfrac{3}{4}(1 + \cos\theta)/b = (\tfrac{3}{4}\cos\theta/b + 11/16b) \neq 0
\end{aligned}
$$

for arbitrary θ.

4. Discussion

We have failed to remove the ghost pole from the spinor–spinor scattering amplitude in lowest order. It is exceedingly unlikely that higher orders will be helpful here, so we conclude that other particles must be present in order to eliminate the ghost. As we showed in I this is a 'massive' spin-2 particle so may be cancelled by a further such particle introduced on its own or as the spin-2 component of a reducible spin-3 state.

At this stage there seems little hope of finding such a state without much further work. However a useful hint may be present in the recent developments of curved super-space (Arnowitt and Nath 1975). Even if curved super-space is not the appropriate theory there may be other gauge theories which have no source terms and also are counter term free at the higher-loop level. Clearly such an avenue has the greatest hope of success for quantizing gravity; the present paper indicates that it is not possible to clear up the mess after the counter terms have been forced into existence.

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